One Belt One Road Conference Series on Number Theory and Combinatorics – I

南京信息工程大学2021年科技活动月—"一带一路"组合与数论研讨会 – I

2021．04．20 – 2021．04．21

Link: https://us02web.zoom.us/j/89996340119

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Program:

Date: 20 of April (Tue)

Time: 9:30-11:45 (CET), 15:30-17:45 (BJT[2])

Chairman: Hao Pan

9:30-9:40 Opening by Wen-Jun Liu

9:40-10:05 Zhi-Wei Sun

10:05-10:30 Lajos Hajdu

10:30-10:55 Yong-Gao Chen

10:55-11:20 András Bazsó

11:20-11:45 Jin-Hui Fang

Date: 21 of April (Wed)

Time: 9:30-11:45 (CET), 15:30-17:45 (BJT)

Chairman: András Bazsó

9:30-9:55 László Szalay & László Németh

9:55-10:20 Hao Pan

10:20-10:45 Gábor Nyul

10:45-11:10 Quan-Hui Yang

11:10-11:35 István Mező

11:35-11:45 Closing

[1]In the Chinese transcription family name comes first

[2]BJT: Beijing Time

Titles and abstracts of the talks

Permutations of $\{1, \ldots, n\}$ and related permanents

Zhi-Wei Sun

In this talk we introduce the speaker's recent results on permutations of $\{1, \ldots, n\}$. For example, we show that for any positive integer n there is a unique permutation $\pi \in S_n$ such that all the numbers $k + \pi(k)$ $(k = 1, \ldots, n)$ are powers of two.

We also mention some divisibility properties of the permanent

$$\mathrm{per}[i^{j-1}]_{1 \le i,j \le n} = \sum_{\sigma \in S_n} \prod_{i=1}^{n} i^{\sigma(i)-1},$$

as well as related applications to groups.

We also introduce some open conjectures of the speaker, one of which states that for any integer $n > 6$ there is a permutation $\pi \in S_n$ such that

$$\sum_{k=1}^{n-1} \frac{1}{\pi(k) + \pi(k+1)} = 1.$$

Perfect powers in arithmetic progressions

Lajos Hajdu

The question that at most how many squares one can find among N consecutive terms of an arithmetic progression, has attracted a lot of attention. An old conjecture of Erdős predicted that this number $P_N(2)$ is at most $o(N)$; it was proved by Szemerédi. Later, using various deep tools, Bombieri, Granville and Pintz showed that $P_N(2) < O(N^{2/3+o(1)})$, which bound was refined to $O(N^{3/5+o(1)})$ by Bombieri and Zannier. There is a conjecture due to Rudin which predicts a much stronger behavior of $P_N(2)$, namely, that $P_N(2) = O(\sqrt{N})$ should be valid. An even stronger form of this conjecture says that we have

$$P_N(2) = P_{24,1;N}(2) = \sqrt{\frac{8}{3}N} + O(1)$$

for $N \ge 6$, where $P_{24,1;N}(2)$ denotes the number of squares in the arithmetic progression $24n + 1$ for $0 \le n < N$. This stronger form has been recently proved for $N \le 52$ by González-Jiménez and Xarles.

In the talk we take up the problem for arbitrary ℓ-th powers. First we characterize those arithmetic progressions which contain the most ℓ-th powers asymptotically. In fact, we can give a complete description, and it turns out that basically the 'best' arithmetic progression is unique for any ℓ. Then we formulate analogues of Rudin's conjecture for general powers ℓ, and we prove these conjectures for $\ell = 3$ and 4 up to $N = 19$ and 5, respectively.

The new results presented are joint with Sz. Tengely.

A conjecture of Sárközy on quadratic residues

Yong-Gao Chen

For any prime p, let R_p be the set of all quadratic residues modulo p. In 2012, Sárközy proved that if p is a sufficiently large prime, then R_p has no 3-decomposition $A+B+C=R_p$ with $|A|,|B|,|C|\geq 2$. In this paper, we prove that for any prime p, R_p has no additive 3-decomposition $A+B+C=R_p$ with $|A|,|B|,|C|\geq 2$. Furthermore, for any prime p, if $A+B=R_p$ is a 2-decomposition, then $0.17\sqrt{p}+1<|A|,|B|<2.8\sqrt{p}-6.63$. This is a joint work with Xiao-Hui Yan.

Polynomial values of sums of hyperbolic binomial coefficients

András Bazsó

In 2016, Belbachir, Németh and Szalay introduced hyperbolic Pascal triangles and hyperbolic binomial coefficients $)^n_k($ as the k-th element in the n-th row of such a triangle. In the talk we investigate power and polynomial values of row sums of hyperbolic Pascal triangles. We present various effective and ineffective finiteness results which are joint work with Lajos Hajdu.

On sets containing no geometric progression with integer ratio

Jin-Hui Fang

Let $\mathbb{N}$ be the set of positive integers and $k\geq 3$ be an integer. Defi

ne the set $G\subset(0,1]$ of real numbers as a *k-good set* if G contains no geometric progression of length k with some ratio $r\in\mathbb{N}\setminus\{1\}$. The real number $x\in(0,1]\setminus G$ is *k-bad with respect to G* if there exists an integer $r\in\mathbb{N}\setminus\{1\}$ such that $G\cup\{x\}$ contains the k-term geometric progression $(x,xr,xr^2,\ldots,xr^{k-1})$. Define

$$Bad(G)=\{x\in(0,1]\setminus G : x \text{ is } k\text{-bad with respect to } G\}.$$

In 2015, M. B. Nathanson and K. O'Bryant proved that there exists a unique strictly increasing sequence $\{A_1^{(k)}<A_2^{(k)}<\cdots\}$ of positive integers with $A_1^{(k)}=1$ such that

$$G^{(k)}=\bigcup_{i=1}^{\infty}\left(1/A_{2i}^{(k)},1/A_{2i-1}^{(k)}\right]$$

is a k-good set and

$$Bad(G^{(k)})=\bigcup_{i=1}^{\infty}\left(1/A_{2i+1}^{(k)},1/A_{2i}^{(k)}\right].$$

They also obtained the values of $A_i^{(k)}$ for $i=1,2,3,4$. Following Nathanson and Bryant's work, we further determined the value of $A_5^{(k)}$.

Generalized Motzkin sequences

László Szalay & László Németh

We consider the Motzkin sequence as the right leg of an arithmetic triangle structurally identical to Pascal's triangle. The sequences located parallel to the Motzkin sequence are called t-generalized Motzkin sequences ($t = 0, 1, 2, \dots$). Three consecutive terms of such a sequence satisfy a linear recurrence of order 2 with quadratic polynomial coefficients. This is simplified to the known recurrence with linear polynomial coefficients when $t = 0$, i.e. in case of Motzkin sequence. In the presentation, we reveal certain properties of this generalization, and some features of the basic triangle.

Two applications of Karlsson-Minton identity on supercongruences

Hao Pan

In this talk, we shall introduce two applications of Karlsson-Minton identity on the supercongruences involving truncated hypergeometric series.

Variations on a theme: generalizations of Bell numbers

Gábor Nyul

Bell numbers are well-known objects in enumerative combinatorics, they count partitions of finite sets. Several generalizations of these numbers can be defined if we forbid certain elements to belong to the same block, or we prescribe restrictions on the cardinality of the blocks, not to mention the Dowling type numbers with abstract algebraic background. In this talk, we give an overview of these directions and our results concerning their combinations.

On the values of representation functions

Quan-Hui Yang

Let $\mathbb{N}$ be the set of nonnegative integers. For a set $A \subseteq \mathbb{N}$, let $R_2(A, n)$ denote the number of solutions to $a + a' = n$ with $a, a' \in A$, $a < a'$. In this talk, we will report some recent results on the values of $R_2(A, n)$ when $R_2(A, n) = R_2(\mathbb{N} \setminus A, n)$ from a certain point on. This is a joint work with Xing-Wang Jiang and Csaba Sandor.

Orders on cohesive and spreading partitions and permutations

István Mező

There is a standard order for set partitions which is based on refinements of partition blocks. This order gives rise to a lattice, originally defined by Birhoff in 1934, with many interesting properties. For permutations, there are two standard orders (and corresponding lattices) one usually encounters with: the strong and weak Bruhat orders. These are specializations of the Bruhat orders from the theory of Coxeter groups.

In our talk we study what is inherited from these lattice structures to situations when we put some restrictions on the partition blocks and permutation cycles.